# Primitive Floats in Coq

Érik Martin-Dorel[1] Pierre Roux[2]
with a lot of work from Guillaume Bertholon

[1]IRIT, Université Paul Sabatier, Toulouse, France

[2]ONERA, Toulouse, France

Thursday 20 June 2019

FEANICSES Workshop

# Proofs involving floating-point computations (1/3)

## Example (Square root)

- To prove that $a \in \mathbb{R}$ is non negative, we can exhibit $r$ such that $a = r^2$ (typically $r = \sqrt{a}$).

# Proofs involving floating-point computations (1/3)

## Example (Square root)

- To prove that $a \in \mathbb{R}$ is non negative,
  we can exhibit $r$ such that $a = r^2$ (typically $r = \sqrt{a}$).
- Using floating-point square root, $a \neq \mathrm{fl}(\sqrt{a})^2$

# Proofs involving floating-point computations (1/3)

## Example (Square root)

- To prove that $a \in \mathbb{R}$ is non negative,
  we can exhibit $r$ such that $a = r^2$ (typically $r = \sqrt{a}$).
- Using floating-point square root, $a \neq \mathrm{fl}(\sqrt{a})^2$
- but one can subtract appropriate (tiny) $c_a$ for which:
  if $\mathrm{fl}(\sqrt{a - c_a})$ succeeds then $a$ is non negative

# Proofs involving floating-point computations (2/3)

## Example (Cholesky decomposition)

- To prove that a matrix $A \in \mathbb{R}^{n\times n}$ is positive semi-definite we can similarly expose $R$ such that $A = R^T R$ (since $x^T \left(R^T R\right) x = (Rx)^T (Rx) = \|Rx\|_2^2 \geq 0$).

# Proofs involving floating-point computations (2/3)

## Example (Cholesky decomposition)

- To prove that a matrix $A \in \mathbb{R}^{n\times n}$ is positive semi-definite we can similarly expose $R$ such that $A = R^T R$ (since $x^T \left(R^T R\right) x = (Rx)^T (Rx) = \|Rx\|_2^2 \geq 0$).
- The Cholesky decomposition computes such a matrix $R$:

$R$ **:=** $0$;
**for** $j$ **from** $1$ **to** $n$ **do**
  **for** $i$ **from** $1$ **to** $j-1$ **do**
    $R_{i,j}$ **:=** $\left(A_{i,j} - \Sigma_{k=1}^{i-1} R_{k,i} R_{k,j}\right) / R_{i,i}$;
  **od**
  $R_{j,j}$ **:=** $\sqrt{M_{j,j} - \Sigma_{k=1}^{j-1} {R_{k,j}}^2}$;
**od**

# Proofs involving floating-point computations (2/3)

## Example (Cholesky decomposition)

- To prove that a matrix $A \in \mathbb{R}^{n \times n}$ is positive semi-definite we can similarly expose $R$ such that $A = R^T R$ (since $x^T \left(R^T R\right) x = (Rx)^T (Rx) = \|Rx\|_2^2 \geq 0$).
- The Cholesky decomposition computes such a matrix $R$:

$R := 0$;
**for** $j$ **from** $1$ **to** $n$ **do**
  **for** $i$ **from** $1$ **to** $j-1$ **do**
    $R_{i,j} := \left(A_{i,j} - \Sigma_{k=1}^{i-1} R_{k,i} R_{k,j}\right) / R_{i,i}$;
  **od**
  $R_{j,j} := \sqrt{M_{j,j} - \Sigma_{k=1}^{j-1} {R_{k,j}}^2}$;
**od**

- With rounding errors $A \neq R^T R$
- but error is bounded and for some (tiny) $c_A \in \mathbb{R}$: if Cholesky succeeds on $A - c_A I$ then $A \succeq 0$.

# Proofs involving floating-point computations (3/3)

## Example (Interval Arithmetic)

- Datatype: interval = pair of (computable) real numbers
- E.g., $[3.1415, 3.1416] \ni \pi$
- Operations on intervals, e.g., $[2,4]-[0,1] := [2-1, 4-0] = [1,4]$, with the enclosure property: $\forall x \in [2,4],\ \forall y \in [0,1],\ x-y \in [1,4]$.
- Tool for bounding the range of functions

# Proofs involving floating-point computations (3/3)

## Example (Interval Arithmetic)

- Datatype: interval = pair of (computable) real numbers
- E.g., $[3.1415, 3.1416] \ni \pi$
- Operations on intervals, e.g., $[2,4] - [0,1] := [2-1, 4-0] = [1,4]$, with the enclosure property: $\forall x \in [2,4],\ \forall y \in [0,1],\ x - y \in [1,4]$.
- Tool for bounding the range of functions
- In practice, interval arithmetic can be efficiently implemented with floating-point arithmetic and directed roundings (towards $\pm\infty$).
- Thus floating-point computations (of interval bounds) can be used to prove numerical facts.

# Motivations

- Coq offers some computation capabilities

$\rightsquigarrow$ which can be used in proofs

- Coq already offers efficient integers

## Goal of this work

- Implement primitive computation in Coq with machine binary64 floats
- Instead of emulating floats with integers (about 1000x slower)

# Agenda

# Agenda

# Coq, computation, and proof by reflection

Coq comes with a primitive notion of computation, called conversion.

**Key feature of Coq's logic: the convertibility rule**

In environment $E$, if $p : A$ and if $A$ and $B$ are convertible, then $p : B$.

# Coq, computation, and proof by reflection

Coq comes with a primitive notion of computation, called conversion.

**Key feature of Coq's logic: the convertibility rule**

In environment $E$, if $p : A$ and if $A$ and $B$ are convertible, then $p : B$.

So we can perform proofs by reflection:

- Suppose that we want to prove $G$.

# Coq, computation, and proof by reflection

Coq comes with a primitive notion of computation, called conversion.

**Key feature of Coq's logic: the convertibility rule**

In environment $E$, if $p : A$ and if $A$ and $B$ are convertible, then $p : B$.

So we can perform proofs by reflection:

- Suppose that we want to prove $G$.
- We reify $G$ and automatically prove that $f(c_1, \ldots) = \text{true} \Rightarrow G$,

# Coq, computation, and proof by reflection

Coq comes with a primitive notion of computation, called conversion.

**Key feature of Coq's logic: the convertibility rule**

In environment $E$, if $p : A$ and if $A$ and $B$ are convertible, then $p : B$.

So we can perform proofs by reflection:

- Suppose that we want to prove $G$.
- We reify $G$ and automatically prove that $f(c_1, \ldots) = \text{true} \Rightarrow G$,
  - by using a dedicated correctness lemma,

# Coq, computation, and proof by reflection

Coq comes with a primitive notion of computation, called conversion.

**Key feature of Coq's logic: the convertibility rule**

In environment $E$, if $p : A$ and if $A$ and $B$ are convertible, then $p : B$.

So we can perform proofs by reflection:

- Suppose that we want to prove $G$.
- We reify $G$ and automatically prove that $f(c_1, \ldots) = \text{true} \Rightarrow G$,
    - by using a dedicated correctness lemma,
    - where $f$ is a computable Boolean function.

# Coq, computation, and proof by reflection

Coq comes with a primitive notion of computation, called conversion.

**Key feature of Coq's logic: the convertibility rule**

In environment $E$, if $p : A$ and if $A$ and $B$ are convertible, then $p : B$.

So we can perform proofs by reflection:

- Suppose that we want to prove $G$.
- We reify $G$ and automatically prove that $f(c_1, \ldots) = \text{true} \Rightarrow G$,
  - by using a dedicated correctness lemma,
  - where $f$ is a computable Boolean function.
  - So we only have to prove that $f(c_1, \ldots) = \text{true}$.

# Coq, computation, and proof by reflection

Coq comes with a primitive notion of computation, called conversion.

**Key feature of Coq's logic: the convertibility rule**

In environment $E$, if $p : A$ and if $A$ and $B$ are convertible, then $p : B$.

So we can perform proofs by reflection:

- Suppose that we want to prove $G$.
- We reify $G$ and automatically prove that $f(c_1, \ldots) = \text{true} \Rightarrow G$,
  - by using a dedicated correctness lemma,
  - where $f$ is a computable Boolean function.
  - So we only have to prove that $f(c_1, \ldots) = \text{true}$.
- We evaluate $f(c_1, \ldots)$.

# Coq, computation, and proof by reflection

Coq comes with a primitive notion of computation, called conversion.

**Key feature of Coq's logic: the convertibility rule**

In environment $E$, if $p : A$ and if $A$ and $B$ are convertible, then $p : B$.

So we can perform proofs by reflection:

- Suppose that we want to prove $G$.
- We reify $G$ and automatically prove that $f(c_1, \ldots) = \text{true} \Rightarrow G$,
  - by using a dedicated correctness lemma,
  - where $f$ is a computable Boolean function.
  - So we only have to prove that $f(c_1, \ldots) = \text{true}$.
- We evaluate $f(c_1, \ldots)$.
- If the computation yields true:

# Coq, computation, and proof by reflection

Coq comes with a primitive notion of computation, called conversion.

**Key feature of Coq's logic: the convertibility rule**

In environment $E$, if $p : A$ and if $A$ and $B$ are convertible, then $p : B$.

So we can perform proofs by reflection:

- Suppose that we want to prove $G$.
- We reify $G$ and automatically prove that $f(c_1, \ldots) = \text{true} \Rightarrow G$,
  - by using a dedicated correctness lemma,
  - where $f$ is a computable Boolean function.
  - So we only have to prove that $f(c_1, \ldots) = \text{true}$.
- We evaluate $f(c_1, \ldots)$.
- If the computation yields true:
  - This means that the type "$f(c_1, \ldots) = \text{true}$" is convertible with the type "$\text{true} = \text{true}$".

# Coq, computation, and proof by reflection

Coq comes with a primitive notion of computation, called conversion.

**Key feature of Coq's logic: the convertibility rule**

In environment $E$, if $p : A$ and if $A$ and $B$ are convertible, then $p : B$.

So we can perform proofs by reflection:

- Suppose that we want to prove $G$.
- We reify $G$ and automatically prove that $f(c_1, \ldots) = \text{true} \Rightarrow G$,
  - by using a dedicated correctness lemma,
  - where $f$ is a computable Boolean function.
  - So we only have to prove that $f(c_1, \ldots) = \text{true}$.
- We evaluate $f(c_1, \ldots)$.
- If the computation yields true:
  - This means that the type "$f(c_1, \ldots) = \text{true}$" is convertible with the type "$\text{true} = \text{true}$".
  - So we conclude by using reflexivity and the convertibility rule.

# Computing with Coq in practice

Three main reduction tactics are available:

1984: `compute`: reduction machine

2004: `vm_compute`: virtual machine (byte-code)

2011: `native_compute`: compilation (native-code)

| method | speed | TCB size |
|---|---|---|
| `compute` | + | + |
| `vm_compute` | ++ | ++ |
| `native_compute` | +++ | +++ |

# Efficient arithmetic in Coq

1994: `positive`, `N`, `Z` $\rightsquigarrow$ binary integers

2008: `bigN`, `bigZ`, `bigQ` $\rightsquigarrow$ binary trees of 31-bit machine integers

- Reference implementation in Coq (using lists of bits)
- Optimization with processor integers in `{vm,native}_compute`
- Implicit assumption that both implementations match

# Efficient arithmetic in Coq

1994: `positive`, `N`, `Z` $\rightsquigarrow$ binary integers

2008: `bigN`, `bigZ`, `bigQ` $\rightsquigarrow$ binary trees of 31-bit machine integers

- Reference implementation in Coq (using lists of bits)
- Optimization with processor integers in `{vm,native}_compute`
- Implicit assumption that both implementations match

2019: `int` $\rightsquigarrow$ unsigned 63-bit machine integers + *primitive computation*

- Compact representation of integers in the kernel
- Efficient operations available for all reduction strategies
- Explicit axioms to specify the primitive operations

# Floating-Point Values

**Definition**

A floating-point format $\mathbb{F}$ is a subset of $\mathbb{R}$. $x \in \mathbb{F}$ when

$$x = m\beta^e$$

for some $m$, $e \in \mathbb{Z}$, $|m| < \beta^p$ and $e_{\mathsf{min}} \leq e \leq e_{\mathsf{max}}$.

# Floating-Point Values

**Definition**

A floating-point format $\mathbb{F}$ is a subset of $\mathbb{R}$. $x \in \mathbb{F}$ when

$$x = m\beta^e$$

for some $m$, $e \in \mathbb{Z}$, $|m| < \beta^p$ and $e_{\text{min}} \leq e \leq e_{\text{max}}$.

- $m$: *mantissa* of $x$
- $\beta$: *radix* of $\mathbb{F}$ (2 in practice)
- $p$: *precision* of $\mathbb{F}$
- $e$: *exponent* of $x$
- $e_{\text{min}}$: minimal exponent of $\mathbb{F}$
- $e_{\text{max}}$: maximal exponent of $\mathbb{F}$

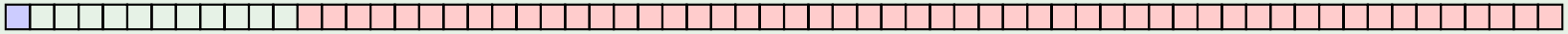
# IEEE 754 standard

The IEEE 754 standard defines floating-point formats and operations.

## Example

For binary64 format (type `double` in C): $\beta = 2$, $p = 53$ and $e_{min} = -1074$.

Binary representation:



sign exponent (11 bits) mantissa (52 bits)

+ Special values: $\pm\infty$ and NaNs (Not A Number, e.g., $0/0$ or $\sqrt{-1}$)

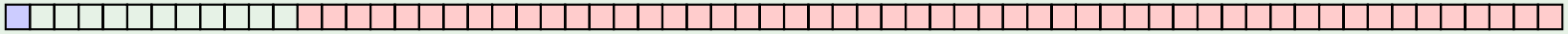
# IEEE 754 standard

The IEEE 754 standard defines floating-point formats and operations.

## Example

For binary64 format (type `double` in C): $\beta = 2$, $p = 53$ and $e_{min} = -1074$.

Binary representation:



sign exponent (11 bits) mantissa (52 bits)

+ Special values: $\pm\infty$ and NaNs (Not A Number, e.g., $0/0$ or $\sqrt{-1}$)

## Remarks

- two zeros: $+0$ and $-0$ ($1/+0 = +\infty$ whereas $1/-0 = -\infty$)

# IEEE 754 standard

The IEEE 754 standard defines floating-point formats and operations.

## Example

For binary64 format (type `double` in C): $\beta = 2$, $p = 53$ and $e_{min} = -1074$.

Binary representation:

sign exponent (11 bits) mantissa (52 bits)

+ Special values: $\pm\infty$ and NaNs (Not A Number, e.g., $0/0$ or $\sqrt{-1}$)

## Remarks

- two zeros: $+0$ and $-0$ ($1/+0 = +\infty$ whereas $1/-0 = -\infty$)
- many NaNs (used to carry error messages)

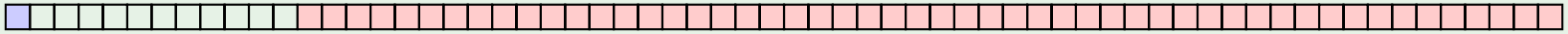
# IEEE 754 standard

The IEEE 754 standard defines floating-point formats and operations.

## Example

For binary64 format (type `double` in C): $\beta = 2$, $p = 53$ and $e_{min} = -1074$.

Binary representation:



sign exponent (11 bits) mantissa (52 bits)

+ Special values: $\pm\infty$ and NaNs (Not A Number, e.g., $0/0$ or $\sqrt{-1}$)

## Remarks

- two zeros: $+0$ and $-0$ ($1/+0 = +\infty$ whereas $1/-0 = -\infty$)
- many NaNs (used to carry error messages)
- $+0 = -0$ but NaN $\neq$ NaN (for all NaN)

# Flocq

Flocq is a Coq library formalizing floating-point arithmetic

- very *generic* formalization (multi-radix, multi-precision)
- linked with *real numbers* of the Coq standard library
- multiple models available
  - without overflow nor underflow
  - with underflow (either gradual or abrupt)
  - IEEE 754 binary format (used in Compcert)
- many classical results about roundings and specialized algorithms
- effective numerical computations

It is mainly developed by Sylvie Boldo and Guillaume Melquiond
and available at http://flocq.gforge.inria.fr/

# CoqInterval

CoqInterval is a Coq library formalizing interval arithmetic

- modular formalization involving Coq signatures and modules
- intervals with floating-point bounds
- radix-2 floating-point numbers (pairs of `bigZ`, no underflow/overflow)

⇝ *efficient* numerical computations

- support of elementary functions such as `exp`, `ln` and `atan`...
- tactics (`interval`, `interval_intro`) to automatically prove inequalities on real-valued expressions.

It is mainly developed by Guillaume Melquiond
and available at http://coq-interval.gforge.inria.fr/

# Agenda

# Workflow

1. Define a minimal working interface for the IEEE 754 binary64 format.
2. Define a fully-specified spec w.r.t. a minimal excerpt of Flocq.
3. Prepare a compatibility layer that could later be added to Flocq.
4. Implementation for `compute`, `vm_compute` and `native_compute`, at the OCaml and C levels.
5. Run some benchmarks.

# Interface (1/4)

```
Require Import Floats.

(* contains *)

Parameter float : Set.
Parameter opp : float → float.
Parameter abs : float → float.

Variant float_comparison : Set :=
  | FEq | FLt | FGt | FNotComparable.
Variant float_class : Set :=
  | PNormal | NNormal | PSubn | NSubn | PZero | NZero
  | PInf | NInf | NaN.
Parameter compare : float → float → float_comparison.
Parameter classify : float → float_class.
```

# Interface (2/4)

```
Parameters mul add sub div : float → float → float.
Parameter sqrt : float → float.
(* The value is rounded if necessary. *)
Parameter of_int63 : Int63.int → float.
(* If input inside [0.5; 1.) then return its mantissa. *)
Parameter normfr_mantissa : float → Int63.int.
Definition shift := (2101)%int63.  (* = 2*emax + prec *)
(* frshiftexp f = (m, e)
   s.t. m ∈ [0.5, 1) and f = m * 2^(e-shift) *)
Parameter frshiftexp : float → float * Int63.int.
(* ldshiftexp f e = f * 2^(e-shift) *)
Parameter ldshiftexp : float → Int63.int → float.
Parameter next_up : float → float.
Parameter next_down : float → float.
```

# Interface (3/4)

Computes but useless for proofs, we need a specification

```
Inductive spec_float :=
  | S754_zero : bool → spec_float
  | S754_infinity : bool → spec_float
  | S754_nan : spec_float
  | S754_finite : bool → positive → Z → spec_float.

Definition SFopp x :=
  match x with
  | S754_zero sx ⇒ S754_zero (negb sx)
  | S754_infinity sx ⇒ S754_infinity (negb sx)
  | S754_nan ⇒ S754_nan
  | S754_finite sx mx ex ⇒ S754_finite (negb sx) mx ex
  end.
(* ... (mostly borrowed from Flocq) *)
```

# Interface (4/4)

And axioms to link everything

```
Definition Prim2SF : float → spec_float.
Definition SF2Prim : spec_float → float.

Axiom FPopp_SFopp :
  ∀x, Prim2SF (-x)%float = SFopp (Prim2SF x).
Axiom FPmult_SFmult :
  ∀x y, Prim2SF (x * y)%float
        = SF64mult (Prim2SF x) (Prim2SF y).
(* ... *)
```

Not yet implemented:

- `roundToIntegral : mode → float → float`
- `convertToIntegral : mode → float → int`

# Pitfalls

| | |
|---|---|
| NaNs | their *payload* is hardware-dependent<br>$\rightsquigarrow$ this could easily lead to a proof of `False` |
| Comparison | do not use IEEE 754 comparison for Leibniz equality<br>(equates $+0$ and $-0$ whereas $\frac{1}{+0} = +\infty$ and $\frac{1}{-0} = -\infty$) |
| Primitive int63 | are *unsigned* $\rightsquigarrow$ requires some care with signed exponents |
| OCaml floats | are *boxed* $\rightsquigarrow$ take care of garbage collector in `vm_compute`<br>(and unboxed float arrays!) |
| x87 registers | $\rightsquigarrow$ double roundings (particularly with OCaml on 32 bits) |

# Pitfalls

NaNs their *payload* is hardware-dependent
$\rightsquigarrow$ this could easily lead to a proof of `False`

Comparison do not use IEEE 754 comparison for Leibniz equality
(equates $+0$ and $-0$ whereas $\frac{1}{+0} = +\infty$ and $\frac{1}{-0} = -\infty$)

Primitive int63 are *unsigned* $\rightsquigarrow$ requires some care with signed exponents

OCaml floats are *boxed* $\rightsquigarrow$ take care of garbage collector in `vm_compute`
(and unboxed float arrays!)

x87 registers $\rightsquigarrow$ double roundings (particularly with OCaml on 32 bits)

Parsing and pretty-printing

- easy solution: hexadecimal (e.g., `0xap-3`)
- ugly and unreadable for humans $\rightsquigarrow$ decimal (e.g., `1.25`)
- indeed, using 17 digits guarantees $parse \circ print$ to be the identity over binary64 (despite $parse$ not injective)
- decimal notations available in Coq 8.10

# Agenda

# Benchmarks (1/3)

[Demo]

- Measure the elapsed time with/without primitive floats for a reflexive proof tactic "`posdef_check`".

| Source | Emulated floats | Primitive floats | Speedup |
|---|---|---|---|
| mat050 | 0.158s ±2.0% | 0.008s ±0.0% | 19.8x |
| mat100 | 1.162s ±1.3% | 0.055s ±5.8% | 21.1x |
| mat150 | 3.605s ±1.2% | 0.176s ±2.2% | 20.5x |
| mat200 | 8.684s ±0.2% | 0.407s ±1.0% | 21.3x |
| mat250 | 17.143s ±1.3% | 0.801s ±0.3% | 21.4x |
| mat300 | 30.005s ±1.2% | 1.366s ±0.7% | 22.0x |
| mat350 | 48.310s ±1.3% | 2.146s ±0.1% | 22.5x |
| mat400 | 70.193s ±1.4% | 3.182s ±0.5% | 22.1x |

- We'd also like to measure the speed-up so obtained on the individual arithmetic operations!

# Benchmarks (2/3) – vm_compute

| Op | Source | Emulated floats CPU times (Op×2−Op) | Op time | Primitive floats CPU times (Op×1001−Op) | Op time | Speedup |
|---|---|---|---|---|---|---|
| add | mat200 | 10.783±0.9% − 8.381±2.8% | 2.403s | 15.718±0.5% − 0.446±1.1% | 0.015s | 157.3x |
| add | mat250 | 21.463±1.7% − 16.405±1.5% | 5.058s | 30.622±0.6% − 0.818±0.6% | 0.030s | 169.7x |
| add | mat300 | 37.430±1.4% − 28.630±1.4% | 8.799s | 53.122±2.4% − 1.400±0.5% | 0.052s | 170.1x |
| add | mat350 | 59.420±0.8% − 45.945±2.9% | 13.475s | 84.194±0.8% − 2.190±0.5% | 0.082s | 164.3x |
| add | mat400 | 87.783±0.9% − 66.173±1.7% | 21.610s | 127.562±8.5% − 3.214±0.3% | 0.124s | 173.8x |
| mul | mat200 | 12.212±1.4% − 8.381±2.8% | 3.831s | 16.096±3.0% − 0.446±1.1% | 0.016s | 244.8x |
| mul | mat250 | 24.517±1.4% − 16.405±1.5% | 8.112s | 31.118±3.7% − 0.818±0.6% | 0.030s | 267.7x |
| mul | mat300 | 42.844±1.7% − 28.630±1.4% | 14.214s | 53.249±0.8% − 1.400±0.5% | 0.052s | 274.1x |
| mul | mat350 | 68.228±1.5% − 45.945±2.9% | 22.283s | 84.332±0.7% − 2.190±0.5% | 0.082s | 271.3x |
| mul | mat400 | 99.722±1.5% − 66.173±1.7% | 33.549s | 125.742±0.8% − 3.214±0.3% | 0.123s | 273.8x |

Table: Computation time for individual operations obtained by subtracting the CPU time of a normal execution from that of a modified execution where the specified operation is computed twice (resp. 1001 times). Each timing is measured 5 times. The table indicates the corresponding average and relative error among the 5 samples (using `vm_compute`).

# Benchmarks (3/3) – native_compute

| Op | Source | Emulated floats<br>CPU times (Op×2−Op) | Op time | Primitive floats<br>CPU times (Op×1001−Op) | Op time | Speedup |
|---|---|---|---|---|---|---|
| add | mat200 | 2.243±1.4% – 1.780±1.7% | 0.463s | 17.681±1.4% – 0.221±0.9% | 0.017s | 26.5x |
| add | mat250 | 4.486±4.2% – 3.411±3.1% | 1.075s | 34.290±0.7% – 0.368±1.5% | 0.034s | 31.7x |
| add | mat300 | 7.249±1.2% – 5.825±4.6% | 1.424s | 59.565±2.5% – 0.553±0.9% | 0.059s | 24.1x |
| add | mat350 | 11.664±3.8% – 9.275±3.5% | 2.389s | 93.818±1.1% – 0.816±0.8% | 0.093s | 25.7x |
| add | mat400 | 17.073±2.9% – 13.142±0.9% | 3.930s | 141.973±2.6% – 1.184±0.9% | 0.141s | 27.9x |
| mul | mat200 | 2.478±1.5% – 1.780±1.7% | 0.698s | 17.807±1.1% – 0.221±0.9% | 0.018s | 39.7x |
| mul | mat250 | 4.824±2.4% – 3.411±3.1% | 1.412s | 35.144±2.1% – 0.368±1.5% | 0.035s | 40.6x |
| mul | mat300 | 8.413±2.4% – 5.825±4.6% | 2.588s | 60.660±2.2% – 0.553±0.9% | 0.060s | 43.1x |
| mul | mat350 | 13.211±2.4% – 9.275±3.5% | 3.937s | 97.248±1.0% – 0.816±0.8% | 0.096s | 40.8x |
| mul | mat400 | 19.269±1.5% – 13.142±0.9% | 6.127s | 138.607±2.3% – 1.184±0.9% | 0.137s | 44.6x |

Table: Computation time for individual operations obtained by subtracting the CPU time of a normal execution from that of a modified execution where the specified operation is computed twice (resp. 1001 times). Each timing is measured 5 times. The table indicates the corresponding average and relative error among the 5 samples (using `native_compute`).

# Agenda

# Concluding remarks

## Wrap-up

- Implementing machine-efficient floats in Coq's low-level layers
- Focus on binary64 and on portability (IEEE 754, no NaN payloads...)
- Builds on the methodology of primitive integers ($\sim$2x / 31-bit retro.)
- Speedup of at least 150x for addition, 250x for multiplication

# Concluding remarks

## Wrap-up

- Implementing machine-efficient floats in Coq's low-level layers
- Focus on binary64 and on portability (IEEE 754, no NaN payloads...)
- Builds on the methodology of primitive integers (~2x / 31-bit retro.)
- Speedup of at least 150x for addition, 250x for multiplication

## Discussion and perspectives

- on-going pull request `https://github.com/coq/coq/pull/9867`
- investigate if `next_{up,down}` could be emulated (and at which cost)
- nice applications (interval arithmetic with Coq.Interval, other ideas?)

# Thank you!

Questions

?

https://github.com/coq/coq/pull/9867